HEAT TRANSFER FROM THE WALL OF A CHANNEL WITH A POROUS LAYER WHEN IT IS FILTERING LIQUID

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When analyzing the temperature conditions in extended underground collectors, heat inflow from the surrounding mass plays an important part. In this case the heat-exchange model in the case of filtering becomes two-dimensional. The formulation of the problem can be simplified considerably if one assumes the filtering region to be one-dimensional, but in this case on the boundary with an impenetrable mass a boundary condition of the third kind must be satisfied. In all papers in thermal rock physics which follow this procedure [1-3] the problem arises of determining the heat-transfer coefficient at the mass-porous sheet boundary. Since the processes occurring in the media when filtering is taking place are fairly slow, it is possible to use the quasistationary approximation. The problem of the heat transfer from the walls of channels with a covering also arises in chemical technology. Various empirical formulas have been proposed for determining the heat-transfer coefficients, which usually hold within narrow limits of the defining criteria. Calculations using the recommended relations lead to considerable spread in the values of the heat-transfer coefficients. A detailed review of this area is given in [4]. There are no theoretical investigations on calculating the heat-transfer coefficients to the walls of a channel when a liquid is being filtered inside it.

In this paper, to calculate the heat transfer coefficient we will consider a double-layer mode (Fig. 1). We will assume that considerable resistance is concentrated on the wall in the region of a sharp temperature gradient, where obviously, molecular heat transfer occurs. The thickness of this region δ_T is not in general the same as the thickness of the hydrodynamic boundary layer δ_Γ. In the nucleus of the flow, due to random interlocking of the current lines on the covering, the heat transfer coefficients are given by [4-6]

$$\lambda_e/\lambda_l = c_1 + c_2\,\mathrm{Re}_r\,\mathrm{Pr}.$$

If we neglect the longitudinal heat transfer (for long tubes) and assume a steady-state profile for the velocity of the liquid at the input to the heated part, the energy equation for the nucleus of the flow can be written in the form

$$u\partial t/\partial x = (\lambda_e/c_p\gamma)[\partial^2 t/\partial r^2 + (1/r)\partial t/\partial r]$$

or

$$\partial t/\partial x = a[\partial^2 t/\partial r^2 + (1/r)\partial t/\partial r],$$

where $a = \lambda_e/c_p\gamma u$; for $x = 0$, $t = t_0$; $q = \text{const}$ for $r = R - \delta_T$; $\partial t/\partial r = 0$ when $r = 0$. The solution of this equation has the form

$$t(r, x) = t_0 + (qR/\lambda_e)[2ax/R^2 - (1/4)(1 - 2r^2/R^2)] - \sum_{n=1}^{\infty} \frac{2}{\mu_n^2 J_0(\mu_n)} J_0\left(\mu_n \frac{r}{R}\right) \exp\left(-\mu_n^2 \frac{ax}{R^2}\right),$$

where $R - \delta_T \simeq R$, since $\delta_T \ll R$. In the stabilized case, when the thermal layers are closed,

$$t(r, x) = t_0 + (qR/\lambda_e)[2ax/R^2 - (1/4)(1 - 2r^2/R^2)].$$

Hence, the temperature at the boundary of the two zones

$$t_S = t_0 + 2qax/\lambda_e R + (q/4)R/\lambda_e.$$

The mean volume temperature in the stabilized part is

$$\bar{t} = \frac{2}{R^2}\int_0^R t r\,dr = \frac{2}{R^2}\int_0^R \left[t_0 + \frac{2qax}{\lambda_e R} - \frac{qR}{4\lambda_e}\left(1 - \frac{2r^2}{R^2}\right)\right] r\,dr = t_0 + 2q\frac{a}{\lambda_e}\frac{x}{R}.$$

Hence $\bar{t} - t_S = -(1/4)qR/\lambda_e$, $q = 4\lambda_e(t_S - \bar{t})/R$.

The thermal flux for the region near the walls can be written in the form

$$q_w = \lambda_l (t_w - t_S)/\delta_T. \tag{1}$$

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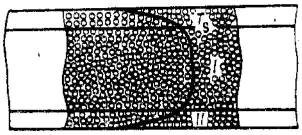

Fig. 1

Equating the thermal fluxes in the first and second zones

$$\lambda_l (t_w - t_S)/\delta_T = 4\lambda_e(t_S - \bar{t})/R,$$

we obtain

$$t_S = [(\lambda_l/\delta_T)t_w + 4\lambda_e\bar{t}/R]/(\lambda_l/\delta_T + 4\lambda_e/R).$$

Substituting the value of t_S in Eq. (1) we find

$$q_w = (t_w - \bar{t})/(R/4\lambda_e + \delta_T/\lambda_l),$$

and for the heat transfer coefficient

$$\alpha = q_w/(t_w - \bar{t}) = 1/(R/4\lambda_e + \delta_T/\lambda_l).$$

It is necessary to determine the quantity δ_T. Our main assumption is the hypothesis that effective transfer begins from the instant when an instability occurs and eddy formation begins when flow occurs round the elements of the coating. In this case the local velocity, which increases on the walls in accordance with the law $u = \tau_w y/\mu$, reaches a certain critical value, so that the Reynolds number corresponding to this velocity, constructed from the dimensions of the particles, becomes critical:

$$(\tau_w \delta_T d_r/\mu)/\nu = \mathrm{Re}^*. \tag{2}$$

We will assume that Re* is constant for all porous coatings with spherical elements. The friction τ_w on the walls of the channel is found in [7] by solving the hydrodynamic problem and has been confirmed experimentally by an electrochemical method. We chose as a basis the Brinkman filtering equation [8], which is the superposition of Darcy's law and the equation for viscous flow in a channel.

It is shown in [7] that when $\mathrm{Re}/\sqrt{K} > 10$, where K is the permeability coefficient of the coating, the friction on the walls of a circular channel is given by

$$\tau_w = \bar{u}\mu/\sqrt{K}. \tag{3}$$

From Eq. (2), using (3), we determine the value of δ_T:

$$\delta_T = \mathrm{Re}^*\nu\mu/\tau_w d_r.$$

Finally we obtain for α

$$\alpha = 1/[\mathrm{Re}^*\nu\sqrt{K(m)}/u\lambda_l + R/4\lambda_e],$$

where $K(m) = m^2/180(1 - m^2)$ is Karman's constant. The effective thermal conductivity λ_e can be found from the equation obtained in [5], according to which, for glass spheres and a radio $d_r/d = 0.12$-0.17:

$$\lambda_e/\lambda_l = 6 + 0.09\,\mathrm{Pr}\,\mathrm{Re}_r. \tag{4}$$

The dimensionless heat-transfer coefficient, taking (4) into account is

$$\mathrm{Nu} = 1/[\mathrm{Re}^*\sqrt{K(m)}/\mathrm{Re} + 1/8(6 + 0.09\mathrm{Pr}\mathrm{Re}_r)]. \tag{5}$$

To check the theoretical model obtained we carried out experimental investigations of the heat transfer using the arrangement shown in Fig. 2.

We investigated the heat-transfer coefficient under conditions of constant thermal flow on the walls. This method of determining the heat-transfer coefficients is much easier and more accurate than the method assuming constant wall temperature employed in [4, 5].

The apparatus was constructed in the form of a circulation contour. The working liquid (water) from a reservoir 6 was applied by means of a centrifugal pump 4 to a tank at a constant level 5. The liquid is returned to the reservoir from the tank through the rotameter 7, the hydrodynamic stabilization section 1 (of length 150 gauges), the experimental section 2, and the mixer 3.

The experimental section consists of a copper tube of internal diameter 15 mm, wall thickness 1.5 mm, and length 370 mm. A nichrome wire is wound with uniform pitch onto the external wall of the experimental part (through a thin layer of insulation). The electric heater is insulated from the external wall with asbestos. The temperature of the walls of

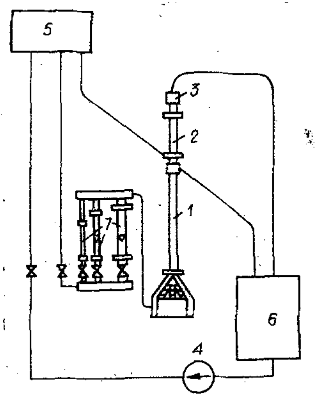

Fig. 2

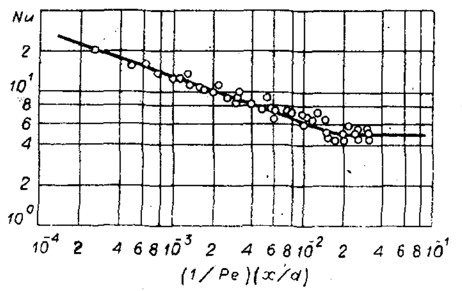

Fig. 3

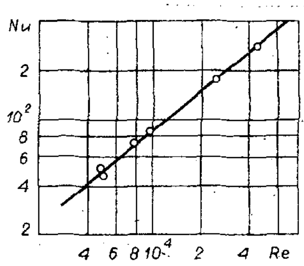

Fig. 4

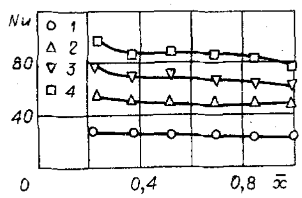

Fig. 5

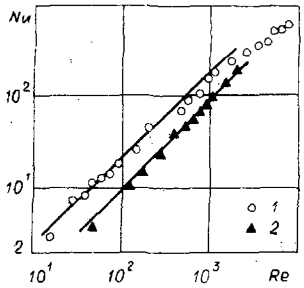

Fig. 6

the tube was measured at eight points along the length using a nichrome–constantan thermocouple laid in special grooves. The heat losses to the surrounding medium were found by calibration experiments when there was no liquid flowing through the working section. To determine the heat losses the temperature of the heater and the surrounding medium were measured in each experiment.

It can be shown that for a constant heat flux at the walls the mean-mass temperature of the liquid varies linearly along the length. Hence, in the experiments we measured the temperature at the input to the experimental section and the calorimetric temperature of the liquid after the mixer at the output.

We first carried out experiments to investigate the heat transfer from the walls of the tube to the flow of liquid in the channel without a coating of spheres.

The experimental data obtained for laminar flow of the liquid are shown in Fig. 3 in the form of curves of Nu = f(x/Pe d). Here the points represent the local values of the dimensionless heat-transfer coefficients for several values of the dimensionless length x/d = 4.3, 7.5, 10.7, 14.2, 17.25, and 20.4, and Re in the range from 100 to 2000, and the line represents a curve of [9]

$$\mathrm{Nu} = 1.31\,[(1/\mathrm{Pe})(x/d)]^{-1/3},$$

which holds for the initial part of the tube with the boundary condition q_w = const. It can be seen that the majority of experimental results lie within the limits of the initial thermal part. There is satisfactory agreement between the experimental and theoretical data. For large values of the relative length (1/Pe) (x/d) the value of the Nu number differs by 5-10% from the theoretical value of the Nu number in the stabilized part.

For turbulent flow the length of thermal stabilization is considerably less, and over the greater part of the tube the heat-transfer coefficient is constant. In Fig. 4 we plot values of Nu in the stabilized thermal part as a function of Re, which shows that the experimental data (the points) agree quite well with the relation [10] $Nu = [(\xi/8)Re\,Pr]/[1{,}07 + 12.7(\sqrt{\xi/8})(Pr^{2/3}-1)]$, which is represented by the line.

The main experiments were carried out when the experimental part was filled with spheres. Two series of experiments were carried out: with glass spheres 3.2 mm in diameter and with polystyrene spheres 1.07 mm in diameter. The permeability coefficient for both forms of coatings was found by the method described in [11]. The change in the dimensionless local heat-transfer coefficient along the length of the tube when there were spheres in the tube is shown in Fig. 5 (1, Re = 250; 2, Re = 360; 3, Re = 680; 4, Re = 880). It can be seen that in this case over a comparatively small distance from the input thermal stabilization begins. Figure 6 shows the Nu number over the stabilized part as a function of the Re number, where the lines denote the values of Nu calculated using Eq. (5) for Re* = 100 (1, d_r = 3.2 mm; 2, d_r = 1.07 mm). Satisfactory agreement between the theoretical and experimental data is observed (Fig. 6).

For a further check of the model it is necessary to carry out experiments over a wider range of Re values and d_r/d ratios.

LITERATURE CITED

1. N. N. Aronova and Yu. P. Morozov, "Mathematical models of heat transfer when a liquid flows in an overground collector," in: Some Problems of Hydrodynamics and Heat Transfer [in Russian], Izd. ITF SO AN SSSR, Novosibirsk (1976).
2. A. N. Shcherban' et al., Methods of Thermal Calculation of Underground Thermal Reactors [in Russian], Izd. Leningr. Gorn. Inst., Leningrad (1974).
3. L. I. Rubinshtein, Thermal Fields in Petroleum Strata [in Russian], Nedra, Moscow (1972).
4. M. É. Aérov and O. M. Todes, Hydraulic and Thermal Principles of the Operation of Apparatus with Stationary and Boiling Granular Layers [in Russian], Khimiya, Leningrad (1958).
5. S. Jagi and N. Wakao, "Heat and mass transfer from wall to fluid in packed beds," AIChE J., 5, No. 1 (1959).
6. S. I. Kuchanov and L. M. Pis'men, "The effective longitudinal heat transfer of a granular layer," Teor. Osn. Khim. Tekhnol., 1, No. 3 (1967).
7. N. N. Smirnova and V. A. Mukhin, "Heat and mass transfer to the walls of a channel when liquid is filtered in it," in: Physical Processes of Rock Formation [in Russian], No. 5, Izd. Leningr. Gorn. Inst. Leningrad (1978).
8. N. C. Brinkman, "A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles," Appl. Sci. Res., A1, No. 1 (1947).
9. B. S. Petukhov, Heat Transfer and Resistance in the Laminar Flow of Liquids through Tubes [in Russian], Énergiya, Moscow (1967).
10. B. S. Petukhov, L. G. Genin, and S. A. Kovalev, Heat Transfer in Nuclear Power Installations [in Russian], Atomizdat, Moscow (1974).
11. Bivers, Sperry, and Rodens, "Effect of the dimensions of the layer on the flow characteristics and porosity of an arbitrarily packed layer of spheres," Prikl. Mekh., No. 3 (1973).